# 18.022 Practice Problems, 12/04/2013

*Recitation Instructor: Homer Reid*

**1.** (Colley problem 7.3.1) Consider the vector field $\mathbf{F} = xz\,\hat{\mathbf{i}} + yz\,\hat{\mathbf{j}} + (x^2+y^2)\,\hat{\mathbf{k}}$ and the surface $S$ defined by $x^2+y^2+5z=1, z \geq 0$ (with the surface orientation chosen such that the surface normal vector has nonnegative $z$ component).

**(a)** Evaluate the surface integral $\int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{A}$.

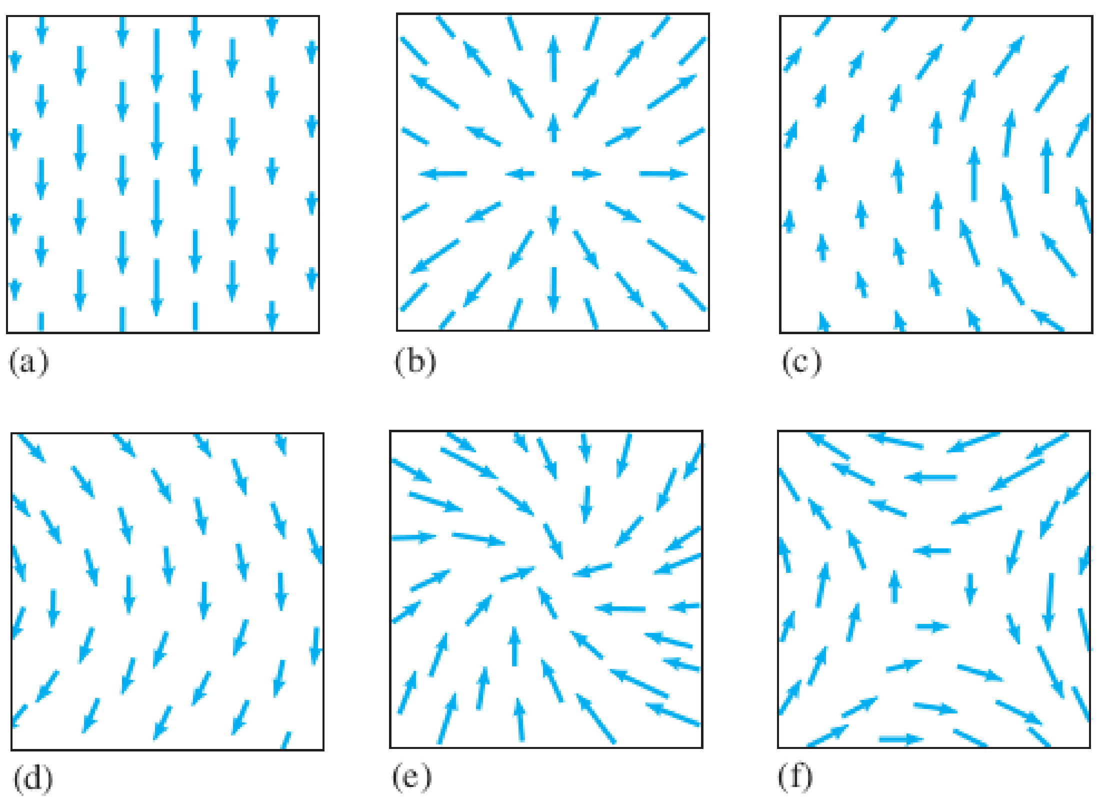
**(b)** Evaluate the line integral $\oint_{\partial S} \mathbf{F} \cdot d\mathbf{s}$.

**2.** (Colley problem 7.3.6) Consider the vector field $\mathbf{F} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ and the subregion $D$ of $\mathbb{R}^3$ defined by the constraint $0 \leq z \leq 9 - x^2 - y^2$.

**(a)** Evaluate the volume integral $\int_D (\nabla \cdot \mathbf{F}) dV$.

**(b)** Evaluate the surface integral $\oint_{\partial D} \mathbf{F} \cdot d\mathbf{A}$.

**3.** (Colley problem 7.3.28) Of the six planar vector fields shown below, four have vanishing divergence at the center of the region depicted, while three have vanishing curl there. Apply reasoning based on Stokes' and Gauss' theorems to categorize each vector field.



**4.** The basic equation of electrostatics relates the electrostatic charge density $\rho(\mathbf{x})$ to the electrostatic field $\mathbf{E}(\mathbf{x})$ as follows:

$$\nabla \cdot \mathbf{E}(\mathbf{x}) = \frac{1}{\epsilon_0}\rho(\mathbf{x})$$

where $\epsilon_0$ is a constant.

Now consider a localized charge density $\rho(\mathbf{x})$ that is known to be fully contained within a spherical region $V$ of radius 2 surrounding the origin. Suppose you measure the electric field at points outside this region and determine that $\mathbf{E}$ takes the form

$$\mathbf{E}(\mathbf{x}) = \frac{3x}{[x^2+y^2+z^2]^{3/2}}\,\hat{\mathbf{i}} + \frac{3y}{[x^2+y^2+z^2]^{3/2}}\,\hat{\mathbf{j}} + \frac{3z}{[x^2+y^2+z^2]^{3/2}}\,\hat{\mathbf{k}}$$

Using this information, determine the total charge $Q = \iiint_V \rho(\mathbf{x})dV$. (The quantity $\epsilon_0$ may appear in your final expression.)[1]

**5.** The basic equation of magnetostatics relates the DC current density $\mathbf{J}(\mathbf{x})$ to the DC magnetic field $\mathbf{B}(\mathbf{x})$ as follows:

$$\nabla \times \mathbf{B}(\mathbf{x}) = \mu_0 \mathbf{J}(\mathbf{x})$$

where $\mu_0$ is a constant.

Now consider a wire oriented along the $z$ axis and and carrying a DC current $I$ . You measure the magnetic field at points on a circle of radius 1 in the $xy$ plane and find that it takes the form

$$\mathbf{B}(x,y,0) = -\frac{7y}{[x^2+y^2]}\,\hat{\mathbf{i}} + \frac{7x}{[x^2+y^2]}\,\hat{\mathbf{j}}.$$

Using this information, determine the value $I$ of the current flowing in the wire. (The quantity $\mu_0$ may appear in your final expression.)[2]

[1] Answer: $Q = 12\pi\epsilon_0$.
[2] Answer: $I = 14\pi/\mu_0$.