18.022 Practice Problems, 12/02/2013

Recitation Instructor: Homer Reid

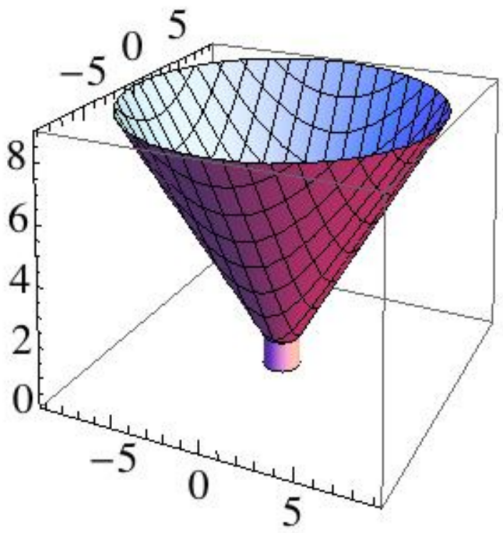
1. (Colley problems 7.2.5 and 7.2.6) Set up and evaluate the following surface integrals.

(a) $\iint_S x^2 dS$ where S is the surface of the cube $[-2,2] \times [-2,2] \times [-2,2]$.

(b) $\iint_S (x^2+y^2) dS$ where S is the lateral surface of the cylinder of radius a and height h whose axis is the z-axis. (Take the lower surface of the cylinder to lie in the xy plane.)

(c) $\iint_S \mathbf{F} \cdot d\mathbf{A}$ where S is the surface of part **(b)** and $\mathbf{F}$ is the electrostatic field of a point charge at the origin, $\mathbf{F}(\mathbf{x}) = \frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}}{(x^2+y^2+z^2)}$.

2. (Colley problem 7.2.23) Let S be the funnel-shaped surface defined by $x^2+y^2=z^2$ for $1 \le z \le 9$ and $x^2+y^2=1$ for $0 \le z \le 1$.



(Figure courtesy of Sam Watson.)

(a) For each point $\mathbf{x}$ on S, determine the outward-pointing surface normal vector to S at $\mathbf{x}$.

(b) Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = -y\hat{\mathbf{i}} + x\hat{\mathbf{j}} + z\hat{\mathbf{k}}$.

3. (Colley problem 7.3.6) Consider the vector field $\mathbf{F} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ and the subregion D of $\mathbb{R}^3$ defined by the constraint $0 \le z \le 9 - x^2 - y^2$.

(a) Evaluate the volume integral $\int_D (\nabla \cdot \mathbf{F}) dV$.

(b) Evaluate the surface integral $\oint_{\partial D} \mathbf{F} \cdot d\mathbf{A}$.