Problem 1. (20 points)

Find the values of the constant k for which the function

$$f(x,y,z) = xyz + kx^2 - kxy + y^2 + kz^2$$

has a non-degenerate local minimum at $(0,0,0)$.

Problem 2. (20 points)

Let $X \subset \mathbb{R}^3$ be the region bounded by the four planes $x = 0$, $y = 0$, $z = 0$, and $x + 2y + 2z = 12$, and let $f: X \to \mathbb{R}$ be the function $f(x,y,z) = xyz$.

(a) Argue that f has a global maximum on X.

(b) Find the maximum of f on X.

Problem 3. (20 points)

a) Draw a picture of the region of integration of $\int_0^1 \int_{2x-1}^{x} dydx$.

b) Change the order of integration to express the integral in part (a) in terms of integration in the order $dxdy$.

Problem 4. (20 points)

Let W be the region in $\mathbb{R}^3$ that lies inside the cone $z^2 = x^2 + y^2$ and between the planes $z = 1$ and $z = 2$.

a) Set up an integral for calculating the volume of the region W.

b) Evaluate the integral in part (a).

Problem 5. (20 points)

Consider the region D in the first quadrant bounded by the curves $y = x^2$, $y = x^2/5$, $xy = 2$, and $xy = 4$.

a) Compute $dxdy$ in terms of $dudv$ where $u = x^2/y$ and $v = xy$.

b) Set up an integral for the area of D in uv-coordinates *and* evaluate it.

1. (20pts) For what values of λ does the function $f: \mathbb{R}^3 \longrightarrow \mathbb{R}$,

$$f(x,y,z) = \lambda x^2 - \lambda xy + y^2 + \lambda z^2,$$

have a non-degenerate local minimum at $(0,0,0)$?

2. (20pts) Let $f: \mathbb{R}^3 \longrightarrow \mathbb{R}$ be the function $f(x,y,z) = x^2 - y^2 + z^2$.
(i) Show that f has a global maximum on the ellipsoid $2x^2+3y^2+z^2 = 6$
(ii) Find this maximum.

3. (20pts)
(i) Switch the order of integration in the integral

$$\int_0^3 \int_{x^2}^9 xe^{-y^2}\, dy\, dx.$$

(ii) Evaluate this integral.

4. (20pts) Let W be the region inside the sphere $x^2 + y^2 + z^2 = 1$ and inside the cone $z^2 = x^2 + y^2$.
Set up an integral to calculate the integral of the function yz over W and calculate this integral.

5. (20pts) Let D be the region in the first quadrant bounded by the curves $x^2 - y^2 = 1$, $x^2 - y^2 = 4$, $xy = 1$ and $xy = 3$.
(i) Find $du\,dv$ in terms of $dx\,dy$, where $u = x^2 - y^2$ and $v = xy$.

(ii) Evaluate the integral

$$\iint_D (x^4 - y^4)\, dx\, dy.$$

1. (20pts) For what values of λ, μ and ν does the function $f\colon \mathbb{R}^3 \longrightarrow \mathbb{R}$,

$$f(x, y, z) = \lambda x^2 + \mu xy + y^2 + \nu z^2,$$

have a non-degenerate local minimum at $(0, 0, 0)$?

2. (20pts) Let $f\colon \mathbb{R}^3 \longrightarrow \mathbb{R}$ be the function $f(x, y, z) = 2x + y - z$
(i) Show that f has a global minimum on the ellipsoid $x^2+2y^2+3z^2 = 6$.
(ii) Find this minimum.

3. (20pts)
(i) Draw a picture of the region of integration of

$$\int_0^1 \int_{1+x}^{\sqrt{9-x^2}} dy\, dx.$$

(ii) Change the order of integration of the integral.

4. (20pts) Let W be the region inside the two cylinders $x^2 + y^2 = 1$ and $y^2 + z^2 = 1$.
Set up an integral to calculate the volume of W and calculate this integral.

5. (20pts) Let D be the region in the first quadrant bounded by the curves $y^2 = x$, $y^2 = 2x$, $xy = 1$ and $xy = 4$.
(i) Find $du\,dv$ in terms of $dx\,dy$, where $u = \frac{y^2}{x}$ and $v = xy$.
(ii) Set up an integral to calculate the area of the region D and calculate this integral.