# 18.022 Practice Problems, 10/30/2013

*Recitation Instructor: Homer Reid*

**1.**

**(a)** Compute the second-order Taylor polynomial for the function $f(x) = \operatorname{atan} x$ at the point $x = 1$.

Note that an alternative way to phrase this problem would be the following: Compute the unique second-order polynomial $P(\Delta) = a_0 + a_1\Delta + a_2\Delta^2$ that agrees most closely with the function $f(\Delta) = \operatorname{atan}(1+\Delta)$ for small values of $|\Delta|$.)

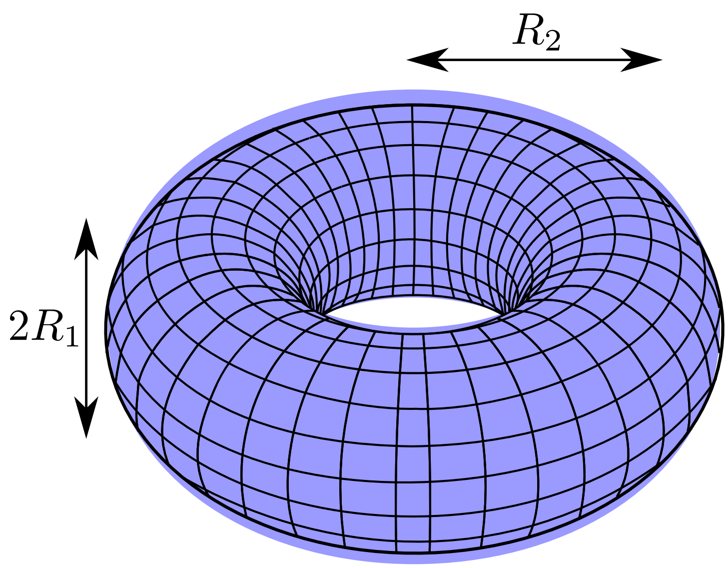
**(b)** Compute the second-order Taylor polynomial for the function $f(x, y) = y \operatorname{atan} x$ at the point $(x, y) = (1, 2)$. Find an alternative description of this polynomial in analogy with the alternative statement of the problem of part **(a)**.

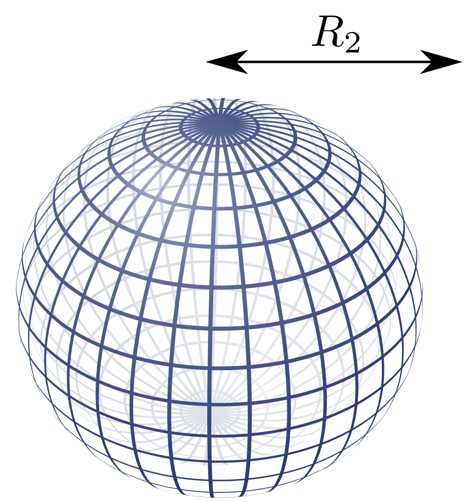
**(c)** Compute the second-order Taylor polynomial for the function $f(x, y, z) = y \operatorname{atan} \frac{x}{z}$ at the point $(x, y, z) = (1, 2, \sqrt{3})$. Find an alternative description of this polynomial in analogy with the alternative statement of the problem of part **(a)**.

**2.** (Colley problem 4.1.20) If you measure the radius of a cylinder to be 2 inches with an error of $\pm 0.1$ inches and the height of the cylinder to be 3 inches with a possible error of $\pm 0.05$ inches, give bounds for the experimental error in your determination of the cylinder's **(a)** volume, **(b)** surface area.

**3.** (Colley, problem 4.2.29) What point on the plane $3x - 4y - z = 24$ is closest to the origin?

**4.** Suppose you are running a donut shop. Your donut is a toroidal volume of fried dough with inner radius $R_1$ and outer radius $R_2$ (measured in centimeters). Each time you make a donut, you also make a "donut hole" (a sphere of radius $R_2$). You sell both the donut and the donut hole together.




The *cost* you incur in making your product is proportional to the quantity of dough you need to make both the donut and the hole (i.e. to the *volume* of the torus plus the volume of the hole). On the other hand, the *price* at which you sell your product is proportional to the *surface area* of the donut plus the surface area of the hole.[1]

Suppose the cost per unit volume of your dough is $\alpha$ (dollars per cubic centimeter), while the price per unit surface area of your product is $\beta$ (dollars per square centimeter). How should you choose $R_1, R_2$ to maximize the profit $P(R_1, R_2)$ you make on each sale?

[1] This is because the larger the surface area of the donut, the more sugar glaze you can slather on it, and customers are willing to pay more for a deeper sugar fix.