# 18.022 Practice Problems, 10/16/2013

Recitation Instructor: Homer Reid

1. Student X wishes to walk from MIT to the Berklee School of Music. The shortest path there would be a straight line down Mass Ave. However, being a citizen of the modern world, Student X must constantly be looking at his phone at all times, especially while walking down the street, and consequently Student X fails to recognize that his route to Berklee in fact traverses the crazy bassackwards non-straight-line trajectory sketched below. (Distances are measured in units of miles.)
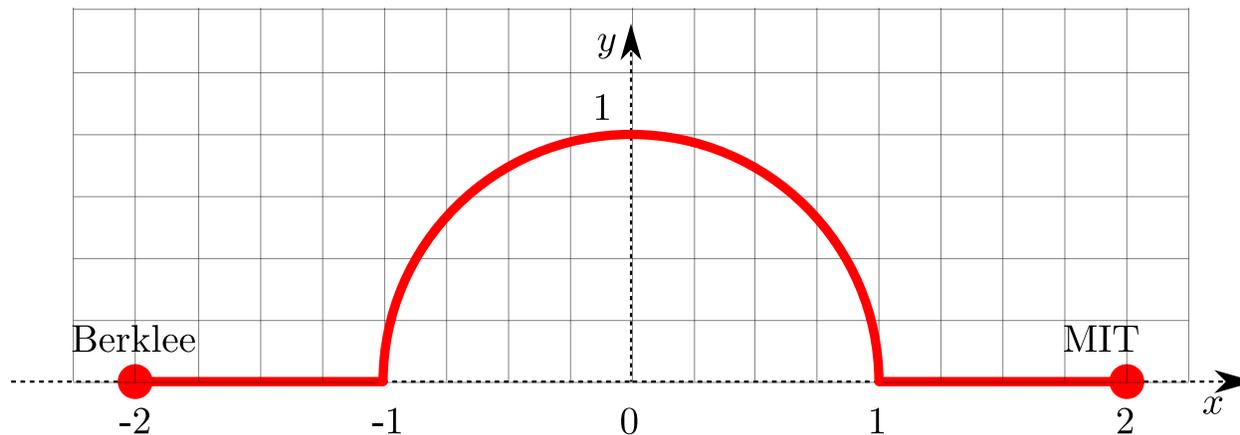

(a) Assuming that Student X begins his journey at time $t = 0$ and walks at a constant speed of 2 miles per hour, write a parametric representation $\{x(t), y(t)\}$ of his coordinates as a function of time. Compute the velocity vector $\mathbf{v}(t)$. Use the arc-length formula to compute the total distance traversed by Student X.

(b) Considered as a curve in $\mathbb{R}^2$, is Student X's trajectory **(i)** continuous, **(ii)** differentiable? If not, at which points does either condition fail to hold?

(c) *Challenge problem.* Now suppose that the circular-arc portion of Student X's journey is deformed into an *elliptical* arc by pushing the north pole of the circle from $(0, 1)$ to $(0, \frac{1}{2})$.
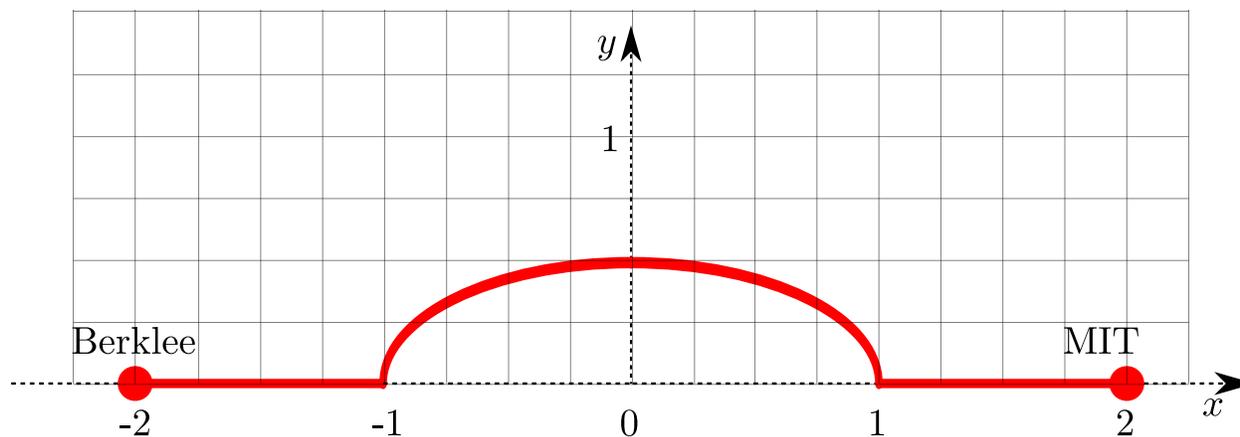
How does your answer to part **(a)** change? (The tricky bit is constructing a trajectory along which the linear velocity remains constant at 2 MPH; it is much easier to construct a trajectory with constant *angular* velocity.)

2. Halfway through his journey to Berklee, Student X is gazing intently into his phone while walking across Commonwealth Avenue on a red light. Shortly thereafter, Student X finds himself mounted vertically to the front grill of an 18-wheel tractor-trailer traveling at 40 MPH toward Boston Common,[1] resulting in the modified trajectory shown below.
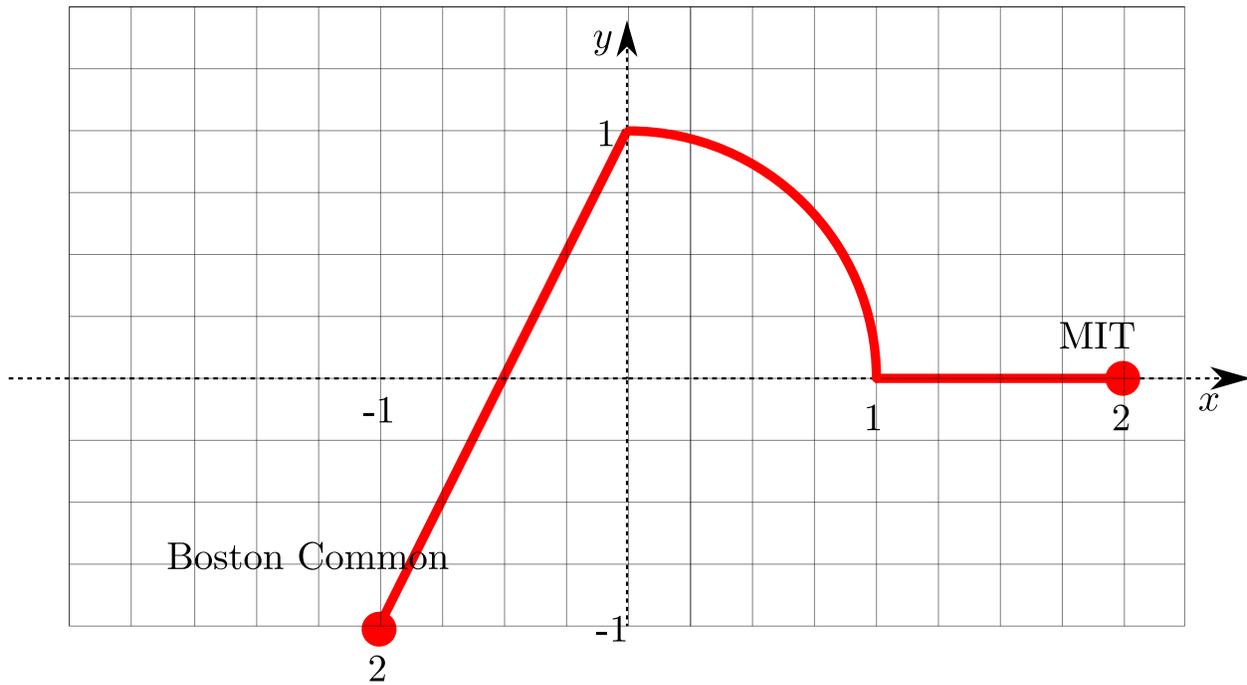
Re-answer all parts of the preceding problem for this modified trajectory. What is the total travel time?

3. Now suppose that the tractor-trailer, which is traveling at 40 MPH when Student X joins its journey, brakes (decelerates) at a constant rate such that its speed reaches 0 when it arrives at Boston Common. What is the travel time for the final leg of the journey?

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[1]Notwithstanding the mechanical challenges of of thumb-typing while serving as a hood ornament, by the time Student X gets to Boston Common he has breathlessly updated his Facebook status to describe his predicament and 17 elementary-school classmates and random cousins from Kansas have "liked" it. 👍