18.022 Practice Problems, 10/09/2013

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1.

(a) Let $\mathcal{V}$ be the subset of $\mathbb{R}^3$ consisting of points (x, y, z) satisfying

$$\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} \le 12.$$

(What is a one-word description of this set of points?) Let $\mathcal{S}$ be the surface (the boundary) of $\mathcal{V}$. Find the outward-pointing normal vector to $\mathcal{S}$ at the point $(x, y, z) = (4, 6, 8)$.

(b) Find a *non-planar* surface in $\mathbb{R}^3$ to which the plane $2x + 3y + 4z = 10$ is tangent at the point $(x, y, z) = (3, 4, -2)$. (Note: There are many possible solutions).

2. Consider the subset of $\mathbb{R}^3$ consisting of points (x, y, z) satisfying the condition

$$x^4 - 18x^2 + 81 + y^4 - 8y^2 + 16 - z^4 = 0.$$

Suppose you are given a point (x_0, y_0, z_0) in this subset. What are the conditions that must be satisfied by (x_0, y_0) to ensure that the function $z(x, y)$ is *locally* well-defined – that is, well-defined for points (x, y) in a neighborhood near (x_0, y_0)?

Note: You can plot the set of points in question using the following MATHEMATICA commands:

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Z[X_, Y_] := Sqrt[Sqrt[(X^2 - 9)^2 + (Y^2 - 4)^2]];
Plot3D[{Z[X, Y], -Z[X, Y]}, {X, -10, 10}, {Y, -10, 10}]
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3. As a generalization of the previous problem, consider the subset of $\mathbb{R}^4$ consisting of points (x, y, z, w) satisfying the simultaneous conditions

$$\begin{aligned} x^4 - 18x^2 + 81 + y^4 - 8y^2 + 16 - z^4 - 2w^4 &= 0 \\ x + y + z + w &= 7 \end{aligned}$$

Suppose you are given a point (x_0, y_0, z_0, w_0) in this subset. What are the conditions that must be satisfied by (x_0, y_0) to ensure that *both* the functions $w(x, y)$ and $z(x, y)$ are *locally* well-defined – that is, well-defined for points (x, y) in a neighborhood near (x_0, y_0)?