# 18.022 Practice Problems, 10/07/2013

*Recitation Instructor: Homer Reid*

**1.** An self-driving car moves in a large test hangar incorporating wind-tunnel devices to test the vehicle's handling during inclement weather. When the wind strength is $W$, the car executes a trajectory such that its coordinates at time $t$ are given by

$$x(t,W) = e^{-W}\big(t^2 + 3t\big), \qquad y(t,W) = e^{-W}\big(3t^3 + 4\big)$$

(here $t$ is measured in seconds and $x, y$ are measured in meters).

The test hangar also features a continuously varying temperature distribution to test the vehicle's climate-control system. The temperature at a point $T(x,y)$ in the hangar is given by

$$T(x,y) = \frac{4x^2}{8-y}$$

[here $x, y$ are measured in meters and $T$ is measured in degrees (Fahrenheit)].

**(a)** First suppose the wind machines are turned off ($W = 0$). How rapidly is the vehicle's temperature changing in time when the vehicle passes through the point $(x,y) = (4,7)$? Express your answer in units of degrees (F) per second.

**(b)** Again with the wind machines turned off, what is the temperature of the vehicle at time $t = 2$? Approximately how much does your answer change if the wind speed is increased to $W = 0.1$?

**2.** Consider the function $z(x,y)$ defined implicitly by the condition

$$xy^2z^2 + \text{atan}(xyz) = 0.$$

(Note that $z = 0$ is a solution of this equation for any $x, y$; here we are interested in solutions satisfying $z \neq 0$.) At the point $(x,y) = (1,2)$, we have $z(x,y) = 0.4168$ (to four decimal places). Approximate the value of $z$ at the point $(x,y) = (1.01, 1.99)$. **(Answer:** $z = 0.4180$.**)**

**3.** (Problem due to Sam Watson.) Consider a conical ice sculpture (picture an upside-down ice-cream cone made of solid ice) of height $h$ and base radius $r$.

**(a)** (Single-variable calculus review problem) Find an expression for the volume $V(r,h)$ of a cone of height $h$ and base radius $r$.

**(b)** Suppose the sculpture is melting in such a way that its height decreases at a rate of 0.001 meters per second while its base radius decreases at a rate of 0.002 meters per second. After the cone has been melting for some time, you measure its height to be $h$ =3 meters and its radius to be $r = 2$ meters. How rapidly is the volume of the cone decreasing at this instant? (*Answer:* $7\pi/750$ $\text{m}^3$/second.)