18.022 Practice Problems, 10/02/2013

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1. Suppose you are at the point $(x, y) = (2, 3)$ in a region whose temperature varies in space according to the following distribution:

$$T(x, y) = \frac{\log x}{1 + y^2} \tag{1}$$

(Following standard metereological practice[1] we are here measuring distance in units of miles.) Now consider the following situations.

(a) You are too cold, and you would like to move to a warmer spot. However, you only have enough gas in your car to drive a distance of 0.1 mile. In what direction should you travel to achieve the greatest possible increase in temperature given this restricted travel distance? Approximately how much warmer will the temperature be at your new destination?

(b) You are perfectly comfortable and do not wish to be warmer or colder. However, you want to go for a 0.1-mile drive. In what direction should you travel to ensure that the temperature remains unchanged?

2. The previous problem was somewhat contrived, because it presumed the existence of an analytical expression [equation (1)] for the temperature distribution. In a more realistic situation, we would not have a nice neat formula for $T(x, y)$, but we might still want to answer questions like **(a)** and **(b)** based on *measurements* of the temperature at various points. To this end, consider the following data:

(i) You measure the temperature at your location to be 40° F. (No wonder you were cold.)

(ii) You call your friend who lives 0.1 mile due east of you (that is, 0.1 mile in the positive x direction), and she informs you that the temperature at her location is 45° F. (Balmy!)

(ii) You call your friend who lives 0.2 mile due south of you (that is, 0.2 mile in the negative y direction), and he informs you that the temperature at his location is 35° F. (And you thought *you* were cold, you wimp.)

Given this information, answer questions **(a)** and **(b)** from Problem 1 above – that is, compute

(a) the direction in which you should travel to achieve the greatest possible increase in temperature

(b) the direction in which you should travel to achieve no change in the temperature.

[1] *Important note:* I actually have no idea what would be standard metereological practice.