18.022 Practice Problems, 9/30/2013

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1. Consider the following three scalar functions of a two-dimensional variable.

$$\textbf{(1)}\quad f_1(x,y)=\sqrt{x^2+y^2},\qquad \textbf{(2)}\quad f_2(x,y)=e^y,\qquad \textbf{(2)}\quad f_3(x,y)=\begin{cases}\text{atan}\,\frac{y}{x}, & x\neq 0\\ 0, & x=0\end{cases}.$$

For each of these functions,

(a) compute the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

(b) Given a point $\mathbf{x}=(x,y)$, think of the partial derivatives of f at $\mathbf{x}$ as the x and y components of a *vector* $\mathbf{v}$, i.e.

$$\mathbf{v}(\mathbf{x})=v_x\hat{\mathbf{i}}+v_y\hat{\mathbf{j}}=\left|\frac{\partial f}{\partial x}\right|_{\mathbf{x}}\hat{\mathbf{i}}+\left|\frac{\partial f}{\partial y}\right|_{\mathbf{x}}\hat{\mathbf{j}}.$$

For several points $\mathbf{x}$ lying on the circles of radius 1 and 2 around the origin, compute the vector $\mathbf{v}(\mathbf{x})$ and plot $\mathbf{v}$ as an arrow based at $\mathbf{x}$.

2.

(a) Consider the curve in $\mathbb{R}^2$ defined by the equation $y=-x^2+6x$. Find an equation for the line tangent to this curve at $x=2$.

(b) Now consider the surface in $\mathbb{R}^3$ defined by the equation $z=-x^2+6x+y^3$. Find an equation for the plane tangent to this surface at $(x,y)=(2,3)$.