18.022 Practice Problems, 9/23/2013

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1. Consider a system of spherical coordinates (ρ, φ, θ) with origin at the center of the Earth and oriented such that the North Pole is at $\varphi = 0$, the equator is at $\varphi = \frac{\pi}{2}$, and the South Pole is at $\varphi = \pi$. (*Note:* We are using the mathematician's convention in which θ is the azimuthal angle and φ is the inclination angle.) Now consider the following pairs of cities. (Think of each city as a single point on the surface of the Earth.)

(i) Boston and Paris, France. **(ii)** Boston and Santiago, Chile.

For each of these pairs of cities, identify which one of statements **(a)**–**(h)** below most accurately describes the relationship between the spherical coordinates of the two cities. (Think of "similar" as meaning "within 25% of each other," while "different" means "differing by more than 25%.")

(a) Different ρ, different φ, different θ.
(b) Different ρ, different φ, similar θ.
(c) Different ρ, similar φ, different θ.
(d) Different ρ, similar φ, similar θ.
(e) Similar ρ, different φ, different θ.
(f) Similar ρ, different φ, similar θ.
(g) Similar ρ, similar φ, different θ.
(h) Similar ρ, similar φ, similar θ.

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2. Consider the unit ball in $\mathbb{R}^3$. (This is the set of all points $\mathbf{x} \in \mathbb{R}^3$ such that the distance from $\mathbf{x}$ to the origin is less than or equal to 1, $|\mathbf{x}| \leq 1$.)

i. Describe this region in spherical coordinates.

ii. Describe this region in cylindrical coordinates.

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3. Solve this problem from the textbook:

39. Consider the solid cylinder pictured in Figure 1.125.

(a) Describe this solid, using cylindrical coordinate (position the cylinder conveniently).

(b) Describe this solid, using spherical coordinates.

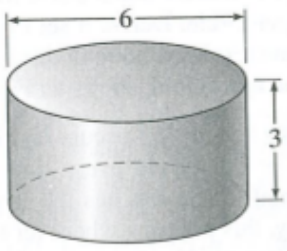

Figure 1.125 The solid cylinder of Exercise 39.

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4. **(a)** Find an equation for the plane containing the following 3 points:

$$(3, -1, 2), \qquad (2, 0, 5) \qquad (1, -2, 4).$$

(b) Find the distance from this plane to the point $P = (8, 0, 3)$.
(c) Find the point P' in the plane that lies closest to P.

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5. **(a)** Consider the surface of revolution obtained by rotating the line $L_1(t) = (t, 2t, t)$ about the z-axis. Describe, in spherical coordinates, the region contained "inside" the resulting cones. **(b)** (Harder) Now repeat this problem for the surface of revolution obtained by rotating $L_1(t)$ about the line $L_2(t) = (t, t, 0)$.