18.022 Practice Problems, 9/11/2013

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1. Consider the following graphical representation of three mutually orthogonal unit vectors in $\mathbb{R}^3$.

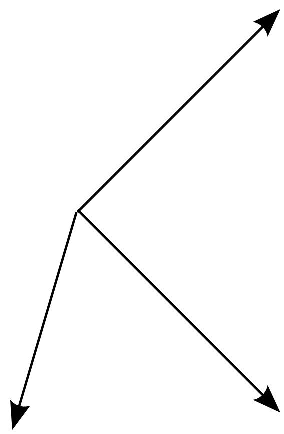

Figure 1: Three mutually orthogonal unit-length vectors in $\mathbb{R}^3$.

Find 3 different ways to assign the labels $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$ to these vectors to ensure that they define a right-handed coordinate system, i.e. we have $\hat{\mathbf{x}} \times \hat{\mathbf{y}} = \hat{\mathbf{z}}$.

2. Consider the triangular prism depicted in the figure below.

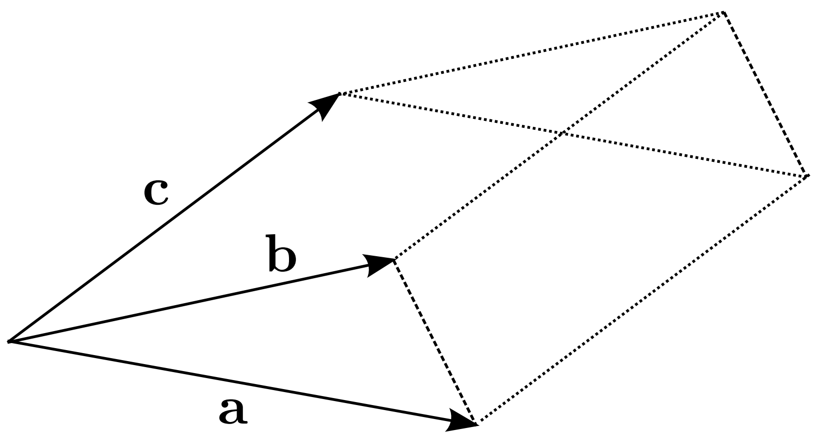

Figure 2: A triangular prism defined by vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$.

Note that, appearances to the contrary notwithstanding, $\mathbf{c}$ is not necessarily normal (perpendicular) to the plane containing $\mathbf{a}, \mathbf{b}$.

(i) Compute the surface area of the prism in terms of $\mathbf{a}, \mathbf{b}, \mathbf{c}$.

(ii) Compute the volume of the prism in terms of of $\mathbf{a}, \mathbf{b}, \mathbf{c}$.

(iii) Suppose I am an unscrupulous jeweler trying to sell bogus pieces to credulous customers. I form a triangular prism out of (cheap) steel, and only coat its surface with (expensive) gold. The larger the volume of the prism, the more I can charge my gullible customers, but the larger the surface area the more it costs me to coat it. How should I choose $\mathbf{a}, \mathbf{b}, \mathbf{c}$ to minimize the ratio of surface area to volume?

For simplicity, you may make the following assumptions:

- $|\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = L$.
- The angle between $\mathbf{a}$ and $\mathbf{c}$ is $\frac{\pi}{4}$.
- The angle between $\mathbf{b}$ and $\mathbf{c}$ is $\frac{\pi}{4}$.

(With these assumptions, the only variable you have to tweak is the angle between $\mathbf{a}$ and $\mathbf{b}$.)