

18.022 Practice Problems, 9/9/2013

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A square of side length 3 centimeters, whose lower-left vertex initially lies at the origin of the xy plane, undergoes a 90° clockwise rotation with its lower-right vertex held fixed.

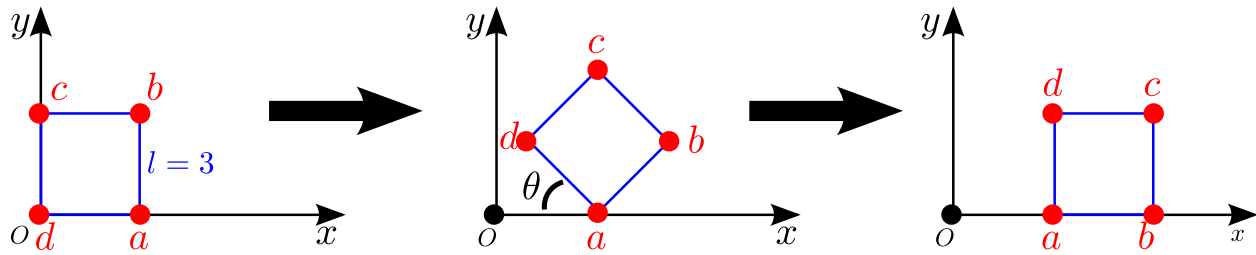


Figure 1: A rotating square.

1. Compute the new coordinates of the points a, b, c, d when the rotation is halfway completed (that is, when $\theta = 45^\circ$ in the figure above).
2. Now generalize this result to obtain parametric equations for the entire curves traced out by points a, b, c, d . Parameterize these curves using the angle θ shown in the figure.
3. Compute, as a function of θ , the distance between point c and the origin.
4. Compute, as a function of θ , the dot product $\vec{ac} \cdot \vec{bd}$. Then compute the *derivative* of this dot product with respect to θ . Interpret this result geometrically.